Optimal Plane Change During Constant Altitude Hypersonic Flight

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In a previous paper, we addressed the problem of choosing constant values of altitude, speed, and angle of attack such that the orbital plane change during hypersonic flight is maximized for a fixed amount of propellant consumption. In the present paper, the restrictions of constant speed and angle of attack are removed. Necessary conditions for solutions to the resulting optimal control problem are derived. Numerical solutions are obtained for several specific cases under the assumption that the constant altitude trajectory that maximizes the plane change is primarily a singular arc. We find that, under the condition of constant altitude flight, it is not, in general, optimum to fly at constant angle of attack. The reduction in plane change capability resulting from a constant angle-of-attack program increases as the range over which the flight takes place increases. On the other hand, the optimum speed is nearly constant.

Introduction

FUTURE spacecraft operating in the vicinity of the Earth may use the atmosphere as an aid in changing orbits. The pioneering work of London[1] established that significant propellant savings are achievable, for certain orbital transfers, by employing a combination of aerodynamic force and propulsive force, rather than relying on propulsive force alone. One example of an "aeroassisted transfer" is the synergetic plane change, in which aerodynamc force is used in part to change the orbital plane of a spacecraft. Two possible flight modes for the atmosphere portion of a synergetic plane change are aeroglide and aerocruise. Aeroglide implies that there is no thrusting. Aerocruise implies that there is thrusting within the atmosphere. Moreover, steady aerocruise implies that the component of thrust along the velocity vector is adjusted to cancel drag, andthereby hold the speed constant, and the vertical components of thrust and lift are used to maintain constant altitude. The lateral components of thrust and lift change the orbital plane. A previous study[2] determined how the orbital plane changes during steady aerocruise and what values of the parameters that define steady aerocruise maximize the plane change. Angle of attack, altitude, and speed were assumed to be constant. Thus the optimization was of a parametric nature. Although the restriction to constant angle of attack, constant speed, and constant altitude flight simplifies the mathematical analysis, this restriction may compromise the performance, i.e., it may be that larger plane changes for a given amount of propellant are possible if these parameters are allowed to vary.

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In the present paper, an intermediate case is considered, namely, constant altitude, variable speed, and variable angle of attack flight. With this generalization, we are faced with an optimal control problem. The controls are taken to be the angle of attack, angle of bank, and thrust magnitude. The problem is to determine the control programs that maximize the plane change achieved for a given amount of propellant consumption. We develop necessary conditions for the optimal controls based on the maximum principle and an analysis of the singular thrust case. We then use the necessary conditions to obtain numerically extremal solutions for the optimal control problem. The solutions are characterized in physical terms, and their optimality is assessed.

Constant Altitude Optimal Control Problem

We shall consider the flight of a thrusting, lifting vehicle of mass m in the atmosphere of a central body. For the purpose of showing clearly the qualitative behavior of the optimal controls, a number of mathematically simplifying assumptions are made. The central body is assumed to be nonrotating with a gravitational field and a stationary atmosphere that depend only on the radial distance from the body's center. The trajectory variables $r,\theta,\phi,V,\gamma,\psi$ are defined in Fig. 1, where r is the radial distance, θ the longitude, ϕ the latitude, V the velocity magnitude, γ the flight-path angle relative to the local horizontal, and ψ the heading angle relative to the local latitude line. We assume that the thrust is aligned with the velocity vector V. Under these assumptions, the equations of translational motion for the center of mass of the vehicle are[3]

$$\frac{\mathrm{d}r}{\mathrm{d}t} = V \sin\gamma \tag{1a}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{V \cos\gamma \cos\psi}{r \cos\phi} \tag{1b}$$

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{V \cos\gamma \sin\psi}{r} \tag{1c}$$

$$m\frac{\mathrm{d}V}{\mathrm{d}t} = T - D - mg \sin\gamma \tag{1d}$$

$$mV\frac{\mathrm{d}\gamma}{\mathrm{d}t} = L \cos\sigma - mg \cos\gamma + \frac{mV^2}{r} \cos\gamma \tag{1e}$$

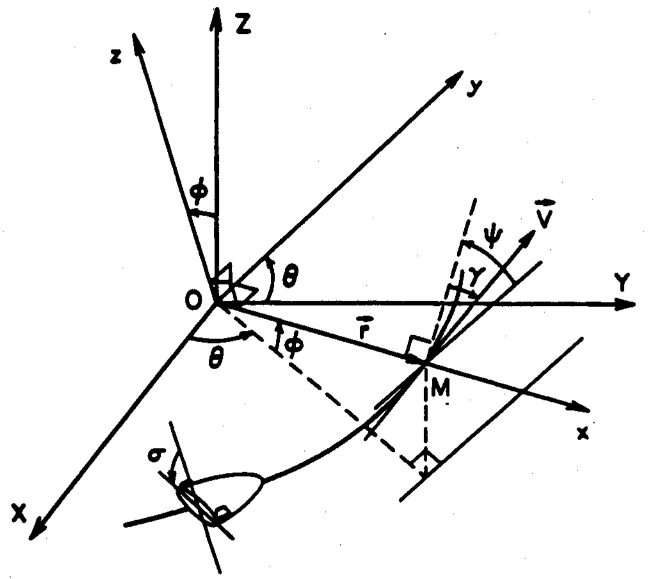

Fig. 1 State variables and bank angle defined with respect to inertial system $OXYZ$.

$$mV\frac{d\psi}{dt} = \frac{L\sin\sigma}{\cos\gamma} - \frac{mV^2}{r}\cos\gamma\cos\psi\tan\phi \tag{1f}$$

$$\frac{dm}{dt} = -\frac{T}{gI_{sp}} \tag{1g}$$

where T is the thrust, D the drag force, L the lift force, σ the bank angle relative to local vertical plane (see Fig. 1), I_{sp} the specific impulse, and g the gravitational acceleration.

The drag and lift forces are given by

$$D = \tfrac{1}{2}C_D S\rho V^2$$

$$L = \tfrac{1}{2}C_L S\rho V^2$$

where C_D and C_L are the drag and lift coefficents, respectively, S a reference surface area, and ρ the atmospheric density. We shall assume a parabolic drag polar for the vehicle

$$C_D = C_{D0} + KC_L^2 \tag{2}$$

where C_{D0} and K are assumed constant for hypersonic velocities. Then, rather than consider angle of attack as a control affecting C_D and C_L, we consider C_L as the control. Denoting the maximum lift-to-drag ratio by $E^* = C_L^*/C_D^*$, it follows from Eq. (2) that

$$C_L^* = \sqrt{C_{D0}/K}$$

$$C_D^* = 2C_{D0}$$

$$E^* = (C_L/C_D)_{\max} = \frac{1}{2\sqrt{KC_{D0}}}$$

Consistent with the assumption of constant altitude flight, we have

$$r \equiv R = \text{const}, \qquad \gamma \equiv 0$$

With altitude constant, it follows from the earlier assumption on the gravitational field that the gravitational acceleration is constant. Since $d\gamma/dt = 0$, we have the relation

$$\frac{1}{2}C_L S\rho V^2\cos\sigma = mg - \frac{mV^2}{r} \tag{3}$$

which shows that the vertical component of the lift force is used to balance the weight minus the centrifugal force. Thus, the lift coefficient (or, equivalently, the angle of attack) and bank angle are not independent controls. Using Eq. (3), we eliminate C_L in the equations of motion, leaving the choice of σ as the sole means of controlling the aerodynamic force.

The dimensionless arc length s will replace the time as the independent variable, where

$$s = \int_0^t \frac{V}{r}\,dt \Rightarrow \frac{ds}{dt} = \frac{V}{r} \tag{4}$$

the following dimensionless variables are introduced

$$u = \frac{V^2}{gR} \tag{5a}$$

$$\mu = \frac{m}{m_0} \tag{5b}$$

The variable u is the square of the ratio of the speed to the circular speed (or the ratio of the centrifugal acceleration to the gravitational acceleration). We will refer to u, henceforth, as the speed. The variable μ is the ratio of the mass at the current value of the arc length to the initial mass. Also, we define the dimensionless parameters and controls as

Dimensionless altitude:

$$Z = \rho SRC_L^*/2m_0 \tag{6a}$$

Dimensionless specific impulse:

$$c = I_{sp}(g/R)^{1/2} \tag{6b}$$

Normalized lift coefficient:

$$\lambda = C_L/C_L^* \tag{6c}$$

Dimensionless thrust:

$$\tau = T/m_0 g \tag{6d}$$

Z is proportional to the density ρ and, hence, it is a parameter defining the flight altitude.

Substituting the relations and parameters just stated into Eqs. (1), we obtain the dimensionless state equations

$$\frac{d\theta}{ds} = \frac{\cos\psi}{\cos\phi} \tag{7a}$$

$$\frac{d\phi}{ds} = \sin\psi \tag{7b}$$

$$\frac{du}{ds} = \frac{2\tau}{\mu} - \frac{uZ}{E^*\mu}\left[1 + \frac{(1-u)^2\mu^2}{Z^2u^2\cos^2\sigma}\right] \tag{7c}$$

$$\frac{d\psi}{ds} = \frac{(1-u)}{u}\tan\sigma - \cos\psi\tan\phi \tag{7d}$$

$$\frac{d\mu}{ds} = -\frac{\tau}{c\sqrt{u}} \tag{7e}$$

The trivial equations for the constant altitude and flight-path angle are not shown. The controls are the bank angle σ and the dimensionless thrust τ, subject to the inequality constraints

$$0 \le \tau \le \tau_{\max} \tag{8}$$

and

$$|\sigma| \le \cos^{-1}\left[\frac{(1-u)\mu}{Z\lambda_{\max}u}\right] \tag{9}$$

At the initial time,

$$s = 0, \quad \theta = 0, \quad \phi = 0, \quad \psi = 0, \quad \mu = 1, \quad u = u_0 \tag{10}$$

At the final time,

$$s_f \text{ free}, \quad \theta_f \text{ free}, \quad u_f \text{ given}, \quad \mu_f \text{ given} \tag{11}$$

Since we want to maximize the inclination angle i_f, we use the performance index

$$J = -\cos i_f = -\cos\phi_f \cos\psi_f \tag{12}$$

We have shown,[2] under the assumptions of a nonrotating central body with an inverse square gravity field and a stationary atmosphere with density a function of radial distance only, that no generality is lost by formulating the maximum inclination change problem with the initial conditions that were stated earlier. What we are doing is redefining longitude and latitude relative to the starting point in the initial orbit plane and maximizing the inclination change with respect to this plane, i.e., maximizing the wedge angle. This is the first step of a two-step optimization procedure[2]; the second step is analytic and involves a transformation to coordinates relative to a distinguished plane (such as the equatorial plane). Regarding the final conditions, we will examine how the maximum inclination angle i_f varies with μ_f. By specifying the final speed u_f, we ensure that the variation is continuously differentiable.

Necessary Conditions for the Optimal Controls

The maximum principle[4] states that, if the control functions τ and σ, piecewise continuous on the interval $[0,s_f]$, are maximizing, then there exists a nontrivial adjoint vector defined on the same interval whose components P_θ, P_ϕ, P_u, P_ψ, and P_μ satisfy the differential equations

$$\frac{dP_\theta}{ds} = 0 \tag{13a}$$

$$\frac{dP_\phi}{ds} = -P_\theta \frac{\cos\psi}{\cos^2\phi}\sin\phi + P_\psi \frac{\cos\psi}{\cos^2\phi} \tag{13b}$$

$$\frac{dP_\psi}{ds} = P_\theta \frac{\sin\psi}{\cos\phi} - P_\phi \cos\psi - P_\psi \sin\psi \tan\phi \tag{13c}$$

$$\frac{dP_u}{ds} = \frac{P_u Z}{E^*\mu}\left[1 - \frac{(1-u^2)\mu^2}{Z^2u^2\cos^2\sigma}\right] + \frac{P_\psi}{u^2}\tan\sigma - \frac{P_\mu \tau}{2cu^{3/2}} \tag{13d}$$

$$\frac{dP_\mu}{ds} = -\frac{P_u u Z}{E^*\mu^2}\left[1 - \frac{(1-u)^2\mu^2}{Z^2u^2\cos^2\sigma}\right] + \frac{2\tau P_u}{\mu^2} \tag{13e}$$

and the transversality conditions

$$P_\theta(s_f) = 0 \tag{14}$$

$$P_\phi(s_f) = \sin\phi_f \cos\psi_f \tag{15}$$

$$P_\psi(s_f) = \cos\phi_f \sin\psi_f \tag{16}$$

and the controls maximize, subject to the constraints (8) and (9), the Hamiltonian

$$H = P_\theta \frac{\cos\psi}{\cos\phi} + P_\phi \sin\psi - \frac{P_u u Z}{E^*\mu}\left[1 + \frac{(1-u)^2\mu^2}{Z^2u^2\cos^2\sigma}\right] + P_\psi\left[\frac{(1-u)}{u}\tan\sigma - \cos\psi\tan\phi\right] + \frac{2\tau}{\mu\sqrt{u}}\left[\sqrt{u}P_u - \frac{\mu P_\mu}{2c}\right] \tag{17}$$

at each point along the optimal trajectory. Since s_f is free, the condition

$$H(s_f) = 0 \tag{18}$$

must also be satisfied. These conditions are necessary but not sufficient. Trajectories and associated controls satisfying these conditions and Eqs. (7), (10), and (11) will be called extremal.

Integrals of the Motion

There are four integrals of the motion, i.e., four relations among the state and adjoint variables that hold along extremal trajectories for the optimal control problem. Because of the fact that $dH/ds = \partial H/\partial s = 0$ and the final condition on H, we have the Hamiltonian integral

$$H = C_0 = 0 \tag{19}$$

As a consequence of the assumed spherical symmetry, we have the integrals[3]

$$P_\theta = C_1 \tag{20a}$$

$$P_\phi = C_2 \sin\theta - C_3 \cos\theta \tag{20b}$$

$$P_\psi = C_1 \sin\phi + (C_2\cos\theta + C_3\sin\theta)\cos\phi \tag{20c}$$

From the transversality condition $P_\theta(s_f) = 0$, it follows that

$$C_1 = 0$$

Optimal Controls

Because of the form of H, the maximization with respect to τ is independent of the maximization with respect to σ and vice versa. Defining the switching function as

$$S = \sqrt{u}P_u - \frac{\mu P_\mu}{2c} \tag{21}$$

the Hamiltonian as a function of τ is maximized according to the rules:

Boost arc:

$$\text{If} \quad S > 0, \quad \text{then} \quad \tau = \tau_{max} \tag{22a}$$

Singular arc:

$$\text{If} \quad S = 0, \quad \text{then} \quad \tau = \text{variable} \tag{22b}$$

Coast arc:

$$\text{If} \quad S < 0, \quad \text{then} \quad \tau = 0 \tag{22c}$$

In the singular case, $S = 0$ over a finite interval in s, the maximum principle does not yield the control τ.

With respect to the bank angle, the Hamiltonian is maximized when $\sigma = \pm\sigma_{max}$ or at an interior point. The first-order necessary condition for an interior maximum is

$$\frac{\partial H}{\partial \sigma} = 0$$

which leads to

$$\tan\sigma = \frac{E^* Z P_\psi}{2P_u(1-u)\mu} \tag{23}$$

This general formula is valid for all boost, coast, and singular arcs. For H to be a maximum at a particular bank angle satisfying Eq. (23), we must have

$$\frac{\partial^2 H}{\partial \sigma^2} = -\frac{2P_u(1-u)^2\mu}{E^* Z u \cos^4\sigma} < 0 \tag{24}$$

This requires that $P_u > 0$.

Along a coast arc (C arc) or a singular arc (S arc), the last term in the Hamiltonian vanishes, and we can combine Eqs. (17), (19), (20), and (23) to obtain the following equation for the optimal bank angle along these arcs

$$AX^2 - 2BX + C = 0 \tag{25}$$

where

$$X = \frac{1-u}{u}\tan\sigma \tag{26}$$

and

$$A = (k\cos\theta + \sin\theta)\cos\phi \tag{27a}$$

$$B = (k\cos\theta + \sin\theta)\cos\psi\sin\phi + (\cos\theta - k\sin\theta)\sin\psi \tag{27b}$$

$$C = -\frac{Z^2}{\mu^2}\left[1 + \frac{(1-u)^2\mu^2}{Z^2u^2}\right](k\cos\theta + \sin\theta)\cos\phi \tag{27c}$$

and where

$$k = C_2/C_3 \tag{28}$$

is a constant to be determined.

Along an S arc, we have over a finite s interval

$$P_u = \frac{\mu P_\mu}{2c\sqrt{u}} \tag{29}$$

Taking the derivative of this equation and using the state and adjoint equations, we obtain

$$\frac{P_\psi\tan\sigma}{u^2} = \frac{ZP_u}{2E^*\mu}\left\{\frac{(1-u)\mu^2}{Z^2u^2\cos^2\sigma}\left[(u+3) + (1-u)\frac{\sqrt{u}}{c}\right] - \left(1 + \frac{\sqrt{u}}{c}\right)\right\} \tag{30}$$

Substituting Eq. (23) into Eq. (30), we obtain the optimal bank angle control for the S arc

$$\left(\frac{\sqrt{u}}{c} - 1\right)X^2 = \frac{Z^2}{\mu^2}\left(\frac{\sqrt{u}}{c} + 1\right) - \frac{(1-u)}{u^2}\left[(u+3) + (1-u)\frac{\sqrt{u}}{c}\right] \tag{31}$$

There are always two solutions to Eq. (31): one positive and one negative. Going back to Eq. (23), because μ and the adjoint P_u are positive for an interior optimal bank angle, it follows that X should always have the same sign as the adjoint P_ψ. Therefore, in numerical calculations, the sign of P_ψ should always be checked, at each time, to ensure that the vehicle is banking in the correct direction. Except for the determination of the sign, the optimal bank angle is given as a function of the state variables only, i.e., in feedback form.

If we eliminate the bank control X between Eqs. (25) and (31), we have an equation relating the state variables along an S arc

$$B^2\left(\frac{\sqrt{u}}{c} - 1\right)\left\{\frac{Z^2}{\mu^2}\left(\frac{\sqrt{u}}{c} + 1\right) - \frac{(1-u)}{u^2}\left[(u+3) + (1-u)\frac{\sqrt{u}}{c}\right]\right\}$$

$$= A^2\left\{\frac{Z^2}{\mu^2} - \frac{(1-u)}{u^2}\left[(1+u) + (1-u)\frac{\sqrt{u}}{c}\right]\right\}^2 \tag{32}$$

Recall that A and B depend on the parameter k. For a given value of k, this equation defines a four-dimensional hypersurface in the five-dimensional state space (θ,ϕ,ψ,u,μ) on which the S arc lies. By taking the derivative of this equation with respect to s, and using the state equations (7) and the control law (31) for the bank angle, we obtain an equation for the variable thrust control

$$\tau = N/D \tag{33}$$

where

$$D = \frac{Z^2}{\mu^2}(\delta^2 - \delta - 1) + \frac{1}{u^2}\left[(1-u)^2\delta^3 + 3(1-u^2)\delta^2 + (1+3u^2)\delta - (3+u^2)\right] \tag{34}$$

$$\begin{aligned} N = &-\frac{2uZ}{E^*}\left\{\frac{Z^2}{\mu^2} - \frac{1-u}{u^2}\left[(1+u) + (1-u)\delta\right]\right\} \times \left[\delta - \frac{\mu^2(1-u)}{Z^2u^2}(\delta+1)\right] \\ &- \frac{uZ}{2E^*}\left[1 + \frac{\mu^2(1-u)^2}{Z^2u^2} + \frac{\mu^2}{Z^2}X^2\right] \times \left\{\frac{Z^2}{\mu^2}(2-\delta) - \frac{2}{u^2}\left[(1-u^2)\delta^2 + (2u^2 - u + 1)\delta - (1+u^2)\right]\right\} \\ &- \frac{uZ^2(\delta-1)^2}{2\mu(AX-B)}\left[A\sin\psi\tan\phi + (k\sin\theta - \cos\theta)\cos\psi\right]X\left[1 + \frac{\mu^2(1-u)^2}{Z^2u^2} + \frac{\mu^2}{Z^2}X^2\right] \end{aligned} \tag{35}$$

and

$$\delta = \frac{\sqrt{u}}{c} \tag{36}$$

This is the thrust that ensures that along an S arc the vector field given by Eqs. (7) with the optimal bank angle is tangent to the hypersurface, and hence, the S arc does not leave the hypersurface. $X = (1-u)\tan\sigma/u$ is computed from Eq. (31). Note that the thrust is a function of the state variables and the as yet unspecified constant $k = C_2/C_3$.

Goh's Higher-Order Necessary Conditions

If an S arc is to be locally maximizing, it is necessary that the matrix H_{vv}, where $v = (\sigma,\tau)^T$ is the control vector, be singular, i.e., that $\det(H_{vv}) = 0$. This condition is satisfied when the thrust is singular. To gain further confidence that the S arc is locally maximizing, we can consider some higher-order necessary conditions that have been derived by Goh[5] for vector controls on an S arc. These conditions are more conveniently expressed in terms of the control vector $w = (X,\tau)^T$, where X is as defined in Eq. (26). Goh's necessary conditions for controls on an S arc to be maximizing are

$$R_1 \leq 0 \tag{37}$$

$$R_3 \leq 0 \tag{38}$$

$$D = R_1R_3 - R_2^2 \geq 0 \tag{39}$$

where

$$R_1 = \partial^2 H/\partial X^2 = -C_3A/X$$

$$\begin{aligned} R_3 = C_3A\big[&Z^2c^2u^2 - 4Z^2cu^{5/2} - 2Z^2u^3 + 4E^*Zc^2u^2\mu \\ &+ 4E^*Zcu^{5/2}\mu - 15c^2\mu^2 - 6c\sqrt{u}\,\mu^2 + 6c^2u\mu^2 + 8cu^{3/2}\mu^2 \\ &+ c^2u^2\mu^2(1+X^2) - 2cu^{5/2}\mu^2(1+X^2)\big]/(2c^2Xu^4\mu^4) \end{aligned}$$

$$D = -C_3^2A^2(\sqrt{u}-c)^2/(c^2u^2\mu^2)$$
$$-C_3^2A^2\big[Z^2c^2u^2-4Z^2cu^{5/2}-2Z^2u^3+4E^*Zc^2u^2\mu$$
$$+4E^*Zcu^{5/2}\mu-15c^2\mu^2-6c\sqrt{u}\,\mu^2+6c^2u\mu^2+8cu^{3/2}\mu^2$$
$$+c^2u^2\mu^2(1+X^2)-2cu^{5/2}\mu^2(1+X^2)\big]/(2c^2X^2u^4\mu^4)$$

The first condition is the classical Legendre-Clebsch condition for the nonsingular control X and is equivalent to that given in Eq. (24) for the control σ. The second condition is the generalized Legendre-Clebsch condition for the singular control τ. The third condition is an additional condition that derives from Goh's transformation approach in the case of more than one control variable. These conditions can be checked numerically along extremal S arcs.

Application to Synergetic Plane Change

We are now ready to consider a synergetic plane change. In such an orbital transfer, the vehicle deorbits and enters the atmosphere at supercircular speed. In order for the formulation and results presented earlier to be applicable, we consider the atmospheric trajectory to be composed of descent, constant altitude, and ascent segments. During the initial segment, the vehicle descends to the cruise altitude. During the constant altitude segment, the plane of the orbit is changed. During the ascent segment, the vehicle is boosted to the desired orbital altitude. A periapse raise burn at orbital altitude completes the transfer. This form of a synergetic plane change, in which there is a constant altitude segment, corresponds to the case in which a heating rate inequality constraint limits the penetration into the atmosphere. More general treatments of the synergetic plane change[6,7] show that, when the unconstrained optimal trajectory violates the heating rate constraint, the constrained optimal trajectory has a finite segment along the constraint boundary, during which the altitude is approximately constant. A more extensive discussion of optimal synergetic plane change can be found in Ref. 8. In the following, we focus on the optimal execution of the plane change during constant altitude flight. Our objective is to ascertain the qualitative nature of the optimal solution, especially for long duration flight.

Previous results[2] have shown that, for maximum plane change, during constant speed, constant angle-of-attack cruise at a given constant altitude, the optimal speed is finite—subcircular at low altitude and approaching circular as the altitude increases. Thus, we expect that for long duration plane change maneuvers it will be desirable to maintain the speed near this optimal steady solution. Since this will require intermediate thrust levels, it follows that the optimal constant altitude trajectory for high altitude will be composed primarily of an S arc. As we noted earlier, S arcs lie on a four-dimensional hypersurface in the five-dimensional state space. In general, a boost or coast arc will be required at the beginning and end of the S arc in order to satisfy the boundary conditions on the speed, i.e., in order to get onto and back off of the singular hypersurface. In the following, we will determine the optimal constant altitude trajectory under the assumption that it consists primarily of an S arc. We will show that the resulting trajectory and associated controls yield a larger plane change than the optimal steady solution. Subsequently, the optimality of the proposed solution will be discussed further.

We begin by considering the optimal constant altitude trajectory to be a totally S arc. It follows from the initial conditions

$$s(0)=0,\qquad \theta(0)=\phi(0)=\psi(0)=0,\qquad \mu(0)=1 \tag{40}$$

that the initial value of B [see Eqs. (27)] is

$$B(0)=0 \tag{41}$$

Moreover, using the transversality conditions (14-16) and Eqs. (20), we obtain

$$\frac{\sin\phi_f\cos\psi_f}{\cos\phi_f\sin\psi_f}=\frac{k\sin\theta_f-\cos\theta_f}{\cos\phi_f(k\cos\theta_f+\sin\theta_f)} \tag{42}$$

from which it follows that

$$B(s_f)=0 \tag{43}$$

So, at both ends of an S arc, Eq. (25) gives the relation

$$X^2=\frac{Z^2}{\mu^2}\left[1+\frac{(1-u)^2\mu^2}{Z^2u^2}\right] \tag{44}$$

Substituting Eq. (44) into Eq. (31) for the optimal bank angle control gives the relation to be satisfied at the two ends of the S arc

$$\frac{Z^2}{\mu^2}=\frac{1-u}{u^2}\left[(1+u)+(1-u)\frac{\sqrt{u}}{c}\right] \tag{45}$$

By setting $\mu=1$, we obtain a relation for computing the initial dimensionless speed for the S arc

$$Z^2=\frac{1-u_0}{u_0^2}\left[(1+u_0)+(1-u_0)\frac{\sqrt{u}}{c}\right] \tag{46}$$

Similarly, the equation for the final dimensionless speed is

$$\frac{Z^2}{\mu_f^2}=\frac{1-u_f}{u_f^2}\left[(1+u_f)+(1-u_f)\frac{\sqrt{u_f}}{c}\right] \tag{47}$$

We see that u_0 and u_f on an S arc are functions of the given parameters Z, c, and μ_f. In general, the values of u_0 and u_f given by Eqs. (46) and (47) will not equal the specified initial and final speeds in Eqs. (10) and (11), and the optimal trajectory must begin and end with a boost or a coast arc, as appropriate. By choosing u_0 in Eq. (10) consistent with the value given by Eq. (46), we will begin our consideration of the optimal trajectory at the junction of the initial boost or coast arc and the S arc. We will attend to the final condition on u at the end of the next subsection. For now, we will assume that the optimal constant altitude trajectory is a totally S arc and ignore the final condition on u.

Combining Eqs. (26), (41), (44), and (46) gives the initial bank angle for the S arc

$$\tan^2\sigma_0=1+\frac{\sqrt{u_0}}{c}+\frac{1+u_0}{1-u_0} \tag{48}$$

Equation (46) shows that for high-altitude cruise, corresponding to small Z, u_0 is nearly unity; hence, the initial bank angle is nearly 90 deg.

Determining the optimal controls for the S arc, although the constant k is all that remains to be determined, requires numerical computation. We select a reference altitude by selecting a value of Z, an engine characteristic by selecting a value of c, and the vehicle's aerodynamic characteristics by selecting a value of the maximum lift-to-drag ratio E^*. We choose

$$Z=0.080640,\qquad c=0.353612,\qquad E^*=2.387 \tag{49}$$

which are the same values as used in Ref. 2. From the definition of Z [see Eqs. (6)] the selected value of Z corresponds to an altitude of about 75 km for a typical hypersonic vehicle, but it can be a higher altitude for a vehicle with lower wing loading m_0g/S or, conversely, a lower altitude for a vehicle with higher wing loading. With these data, we compute the initial speed for the constant altitude S arc from Eq. (46) and obtain $u_0=0.996779$. The integration of the state equations (7) is performed using the initial conditions (40). The bank angle control is given by Eq. (31); the thrust magnitude is given by Eq. (33). The thrust magnitude depends on the constant k, which must be specified. We use Eq. (32), which is essentially the Hamiltonian integral along an S arc, to check the accuracy of the numerical integration. It should be noted that to avoid numerical error in the evaluation of the bank angle, due to the behavior of the terms $(1-u)\tan\sigma$ and $(1-u)/\cos\sigma$ when $\sigma \approx 90$ deg, we use the definition (26) to express these terms in terms of the control X in the state equations (7).

Maximum Plane Change Singular Arc

The procedure outlined in the preceding paragraph generates an extremal S arc for a given value of k. For a specified final mass ratio μ_f, k is adjusted such that the condition on the final mass ratio and the transversality condition (43) are satisfied simultaneously. However, there are typically several values of k that will work, and, consequently, there are several extremal S arcs that are candidates for maximizing the plane change. We examine Figs. 2 and 3 to gain insight into this situation.

For the selected altitude, two trajectories, corresponding to $k = 1.0$ and 0.5, are plotted in the u-μ plane in Fig. 2. (For positive k values, the vehicle begins turning to the left.) Note that the speed oscillates as the mass decreases, but it is never far from circular speed ($u = 1$). The local minima in the speed occur at the points where a trajectory touches the curve labeled $A = 0$. This curve has the following significance. Referring to Eqs. (20) and (27), we see that A is the same as the adjoint P_ψ except for the nonzero constant factor C_3. Equation (23) indicates that the bank angle is zero when P_ψ, or A, is zero. This occurs when the vehicle is near an apex of the osculating orbit and the bank angle is midway through a continuous, but quick, transition from $+90$ to -90 deg or vice versa, i.e., midway through a bank reversal. For $\sigma = 0$, we have $X = 0$; consequently, Eq. (31) reduces to a relation between the dimensionless speed u and the mass ratio μ at the bank reversal points, namely,

$$\frac{Z^2}{\mu^2}\left(\frac{\sqrt{u}}{c}+1\right) = \frac{1-u}{u^2}\left[(3+u)+(1-u)\frac{\sqrt{u}}{c}\right] \tag{50}$$

This relation gives rise to the curve labeled $A = 0$. Thus, intersections of this curve and a trajectory indicate the occurrence of a bank reversal.

As stated earlier, the constant k is selected such that, when $B = 0$, the prescribed final mass ratio is reached. We have also determined that $B = 0$ at the beginning of an S arc. Combining Eqs. (25) and (31), we obtain

$$B\left(\frac{\sqrt{u}}{c}-1\right)X = A\left\{\frac{Z^2}{\mu^2} - \frac{1-u}{u^2}\left[(1+u)+(1-u)\frac{\sqrt{u}}{c}\right]\right\} \tag{51}$$

When $B = 0$, A is not necessarily zero. Hence, we deduce that when $B = 0$ we have

$$\frac{Z^2}{\mu^2} = \frac{1-u}{u^2}\left[(1+u)+(1-u)\frac{\sqrt{u}}{c}\right] \tag{52}$$

which is the same as Eq. (45). This relation between u and μ corresponds to the curve labeled $B = 0$ in Fig. 2. Any totally S arc should start and end on the $B = 0$ curve. In the case of a long duration trajectory (corresponding to small μ_f), it may touch the $A = 0$ curve a few times and cross the $B = 0$ curve a few times. For a short trajectory, however, it may not touch the $A = 0$ curve at all, i.e., there may be no bank reversals.

The value of k that yields, for each value of the final mass ratio, the maximum plane change within in the class of pure S arc trajectories can be determined from a direct performance comparison between the candidate extremal solution. The graph of the plane change achieved via an extremal S arc as a function of the final mass ratio, where at the final point $B = 0$

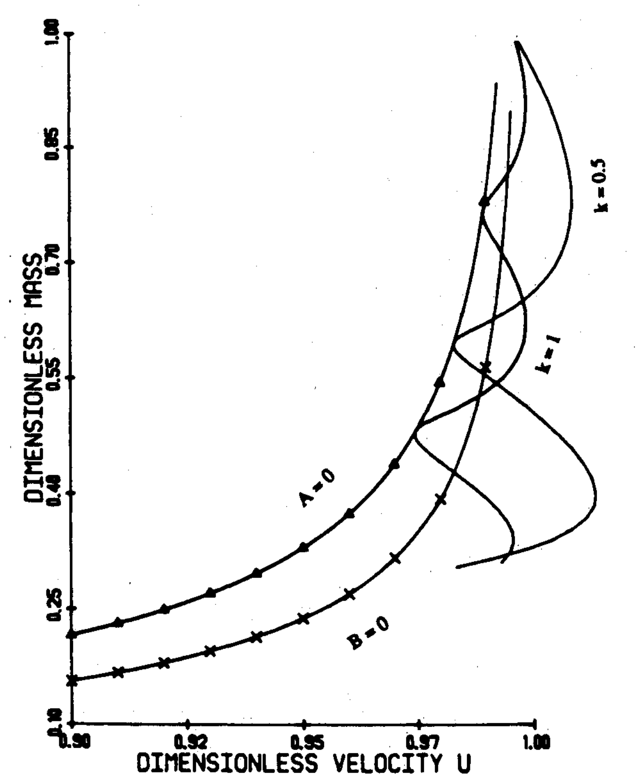

Fig. 2 Dimensionless velocity u vs mass μ history and crossing points ($Z = 0.080640$).

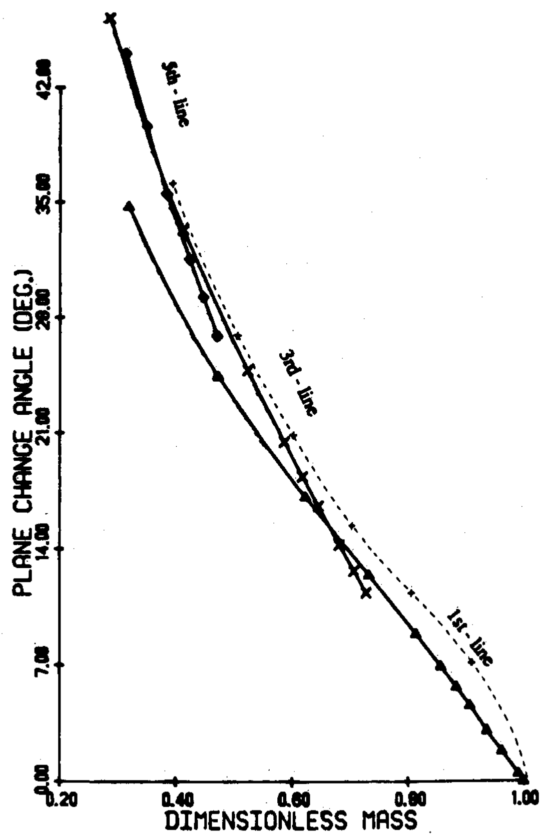

Fig. 3 Optimal plane change angle i ($Z = 0.080640$).

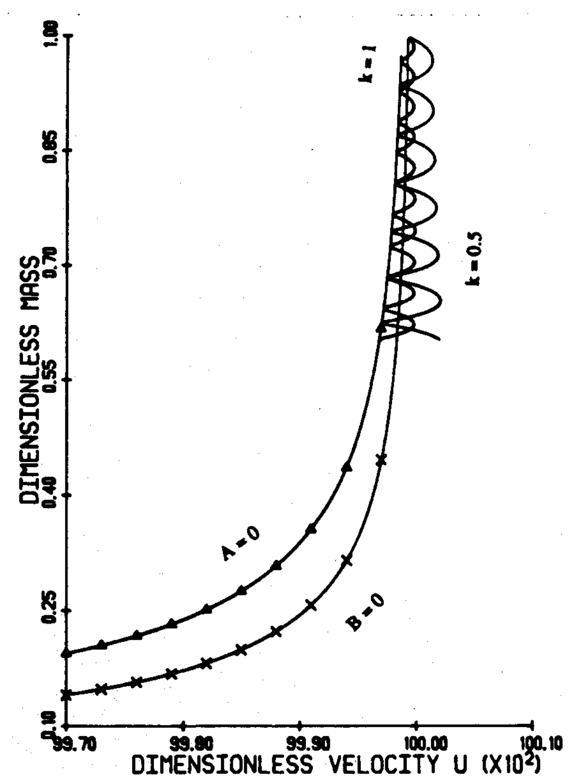

Fig. 4 Dimensionless velocity u vs mass μ history and crossing points ($Z = 0.010913$).

for the first time (not counting the initial point), is labeled first line in Fig. 4. Each point on this curve indicates the plane change and final mass ratio for an extremal arc corresponding to some value of k. The optimal bank angle does not depend explicitly on k; the optimal thrust does. As k decreases, the thrust level increases and final mass ratio (when $B = 0$) decreases. The only term in the expression for the optimal thrust involving k is a homographic function of k, and, thus, as k increases, the thrust law approaches asymptotically a form independent of k.

Similarly, the graph of the plane change achieved via an extremal S arc as a function of the final mass ratio, where at the final point $B = 0$ for the third time, is labeled third line, and so on. Note that, between roughly $\mu_f = 0.32$ and 0.67, selecting a value of k such that the final mass ratio is reached at the point where $B = 0$ for the third time produces the largest plane change. For values above this range, selecting a value of k such that the final mass ratio is reached at the point where $B = 0$ for the first time produces the largest plane change. Just below $\mu_f = 0.32$, the plane change is maximized by selecting a value of k such that the final mass ratio is reached at the point where $B = 0$ for the fifth time. For still lower values of μ_f, the seventh $B = 0$ point is best, and so on.

Connecting the segment of the first line from $\mu_f = 1.0$ to 0.67 to the segment of the third line from $\mu_f = 0.67$ to 0.32, etc., we obtain the graph of the maximum plane change as a function of the final mass ratio. The graph is continuous but its slope has discontinuities at the connection points. This is because we have assumed that the optimal constant altitude trajectory is a totally S arc, and, consequently, the final speed u_f is determined from Eq. (48) and, in general, does not satisfy the prescribed final condition (11). If we require u_f to be the same value for each trajectory, then i_f, as a function of μ_f, will have a continuous first derivative. If the specified common value of u_f is higher than the value achieved on the pure S arc, for a particular trajectory, then a final boost arc is required. In this case, the fuel consumption on the S arc must be less than that corresponding to μ_f in order to leave sufficient fuel for the boost. If the specified value of u_f is lower, then a final coast arc is required. The transversality condition, to be satisfied at the final time, is still $B_f = 0$, but as given by Eqs. (27), rather than by Eq. (52), which is only valid for an S arc.

To illustrate the point made in the preceding paragraph, we select a common value of $u_f = 0.95$ that is always less than the final value achieved on the S arc. Then, for each specified μ_f, we guess the value of k and integrate the equations for a totally S arc as before. When μ attains μ_f with $u > 0.95$, the switch is made to coasting flight, i.e., to $\tau = 0$, but now with the bank control X obtained from Eq. (25). At the final speed $u_f = 0.95$, the transversality condition (43) is checked. This procedure is repeated until the value of k that satisfies the transversality condition is found. Carrying out the computations, we find that the total plane change i_f is now higher, as shown by the dashed line in Fig. 3, and there are no discontinuities in the slope.

Comparison with Optimal Steady Cruise Solution

We are now in a position to compare the optimal constant altitude solution, in which the bank angle (or equivalently, the angle of attack) and the thrust are modulated, to the optimal steady cruise solution, in which the angle of attack and the thrust are constant. We consider the same altitude as previously, for which $Z = 0.080640$, and specify that $\mu_f = 0.6$. The S arc solution gives a plane change of $i_f = 19.7$ deg; the highest lift coefficient required is about $\lambda = 1.2$. For the steady cruise case, the bank angle is given by the equilibrium condition (3), whereas the thrust is set to cancel the drag. Solving the parametric optimization problem to obtain the optimal speed and lift coefficient, we obtain the optimum values $u = 0.998$ and $\lambda = 1.8$. The bank angle increases from the initial value of 89.2 deg to the final value of 89.5 deg. The time of flight is shorter. The resulting plane change is $i_f = 17.6$ deg.

However, the comparison is not complete since the steady cruise speed is slightly higher than both the initial speed $u_0 = 0.9968$ and the final speed $u_f = 0.9912$ for the S arc. These differences are adjusted by adding an initial C arc to the S arc and prolonging the steady turn by a final C arc. Consider the two equations for u and ψ in the new form

$$\frac{du}{ds} = -\frac{uZ(1+\lambda^2)}{E^*\mu} \tag{53a}$$

$$\frac{d\psi}{ds} = \frac{Z\lambda}{\mu}\sin\sigma - \cos\psi\tan\phi \tag{53b}$$

Neglecting the small term in $\tan\phi$, we combine these equations to obtain

$$\frac{d\psi}{du} = -\frac{E^*\lambda\sin\sigma}{(1+\lambda^2)u} \tag{54}$$

The most favorable turn is conducted with $\lambda = 1$ and $\sigma = 90$ deg. Then, by integrating from u_1 to u_2, we have

$$\Delta i \approx \Delta\psi = \frac{E^*}{2}\log\frac{u_1}{u_2} \tag{55}$$

According to this formula, the adjusted plane changes are $i_f = 19.7 + 0.08 = 19.78$ deg for the S arc and $i_f = 17.6 + 0.47 = 18.07$ deg for the steady turn. Although the improvement in plane change capability, attainable through angle of attack and thrust modulation, is small for the altitude considered, it becomes more and more substantial as the altitude increases.

Nature of Optimal Trajectories

In order to emphasize the characteristics of the variable thrust, multirevolution trajectories, we consider a higher altitude corresponding to $Z = 0.010913$. The initial speed for the S arc is $u_0 = 0.999940$, almost circular. If h is the altitude difference with respect to the previous flight level $Z_0 = 0.080640$ taken as reference, and β is the inverse scale height for an exponential atmosphere, the new value of Z satisfies the equation

$$\beta h = \log(Z_0/Z) = 2$$

Hence the new altitude is about 15 km higher. Since the atmospheric density is lower, we expect a longer range trajectory for a given amount of fuel consumption, in comparison to the case considered earlier.

Fig. 4 shows that there is even less deviation from circular speed than there was at the lower altitude, namely, 0.1% vs 3.0%. From the number of intersections with the $A = 0$ line, it is clear that, for a given amount of fuel consumption, more revolutions are required at higher altitude to achieve the plane change, as expected. Figure 5 shows that the optimal number of revolutions increases as the propellant consumption increases.

Representative plots of the optimal controls for the higher altitude case are shown in Figs. 6–8. The value of μ_f is 0.6.

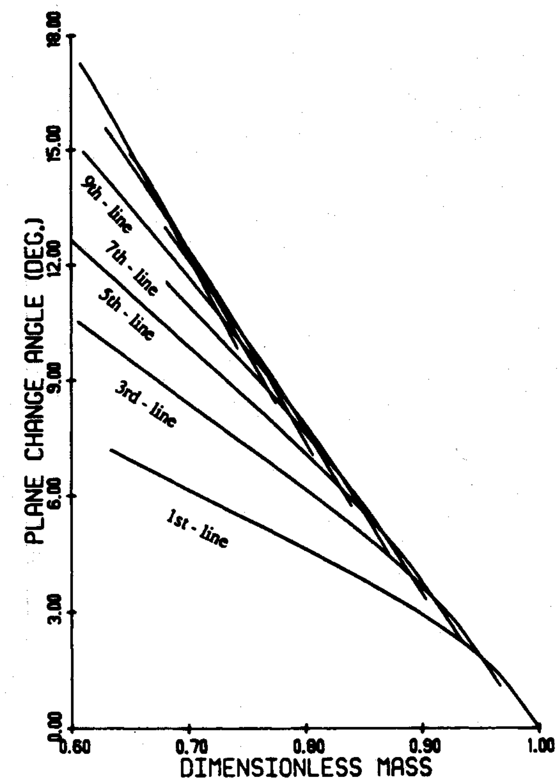

Fig. 5 Optimal plane change angle i ($Z = 0.010913$).

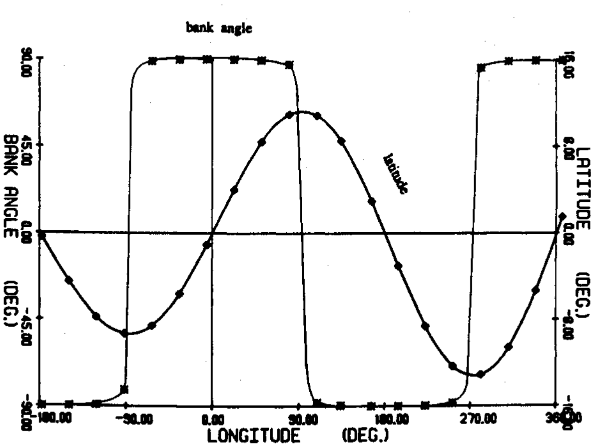

Fig. 6 Comparison between bank angle and latitude along the optimal flight path.

For this case, it takes about five revolutions to achieve the largest plane change via a totally S arc. The optimal plane change is 17.8 deg, which is less than the 19.7 deg achievable in the lower altitude case, for the same fuel consumption. The optimal controls are nearly periodic. In Fig. 6, the latitude and the optimal bank angle are plotted against the longitude. The longitude is reset at $\theta = 0$ at the ascending node $\phi = 0$ for a typical revolution. The increase in the absolute values of the maxima and minima of the latitude shows that the inclination is increasing. The optimal bank angle is nearly bang-bang. The optimal bank angle switches values at the apexes of the orbit. It is $+90$ deg on the side containing the ascending node and -90 deg on the side containing the descending node. A bank angle magnitude of 90 deg ensures that all the lift force is used for turning. The switching of the sign ensures that the inclination continues to increase.

Figure 7 shows the optimal normalized lift coefficient (which is directly related to angle of attack), along with the optimal bank angle, as functions of the longitude. It has been noted that the lift coefficient and the bank angle are related [see Eq. (3)]. Previous work[2,9] has shown that for a given heading angle change the corresponding inclination change is maximized, if the heading change is made at a node. The behavior of the normalized lift coefficient is consistent with this finding. We see that the lift increases to its maximum at the nodes. The reason the lift is not always at its maximum is indicated in Fig. 8. Figure 8 shows the normalized thrust magnitude as a function of the longitude. Since the lift vector is essentially always in the horizontal plane, the altitude is kept constant by flying at nearly circular speed. The control for

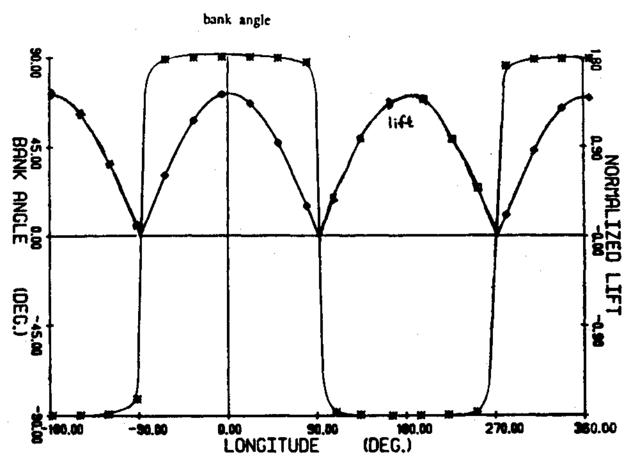

Fig. 7 Comparison between bank angle and normalized lift along the optimal flight path.

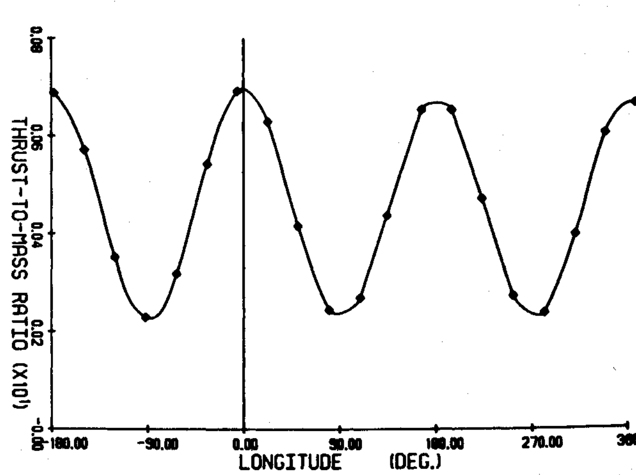

Fig. 8 Variation of thrust along the optimal flight path.

maintaining nearly circular speed is the thrust magnitude. Although maximum lift is good for turning, high lift means high drag. Consequently, the thrust magnitude is seen to peak in phase with the lift. The lift decreases to a minimum at the apexes of the orbit because heading changes lose their effectiveness in changing the inclination and because propellant can be saved. In contrast, the steady cruise turn, because the lift coefficient (angle of attack) and speed are constrained to be constants, does not provide the freedom to compensate for the spatial nonuniformity in the effectiveness of out-of-plane forces to change the orbital plane and, consequently, the performance of the steady cruise turn deteriorates at high altitude.

As a final note, Fig. 8 shows that only a low thrust capability is required to fly the sustaining arc at high altitude and accomplish a substantial plane change.

Optimality Assessment

Under the assumption that the optimal constant altitude trajectory is a totally S arc, i.e., that it lies on the singular hypersurface, we have computed all the extremal solutions and determined by direct comparison the one with the largest plane change. We have been able to determine all of the extremal solutions because an S arc is completely determined by specifying the value of the single constant parameter k and because the behavior of the S arc as a function of k is clearly identifiable from our graphical analysis. Besides the value of k, extremal S arcs are distinguished by the number of times n that the oscillatory function B has passed through zero. The condition $B = 0$ is a transversality condition that must be satisfied at the final time. Locations at which $B = 0$ occur in between an apex and a node of the osculating orbit; hence, there are four occurrences per revolution. (We use the term revolution to refer to travel from one crossing of the initial orbit plane in the ascending direction to the next crossing of the initial orbit plane in the ascending direction. During a revolution, the inertial longitude may change by more or less than 360 deg if the line of nodes is changing.) Consistent with the fact that the effectiveness of a heading change for changing the inclination is maximum at the nodes and minimum at the apexes, we find that the S arc that maximizes the inclination change always ends at an odd-numbered occurrence of $B = 0$. In other words, it is never optimal to reduce the rate of fuel consumption in order to pass through the upcoming apex, unless the flight is continued through the subsequent node as well. Numerical experience indicates that the maximizing S arc is achieved by selecting the largest value of k for which the specified final mass ratio and transversality condition (43) can be achieved with n odd.

Thus, we have a combined graphical-numerical approach for determining, for a given cruise altitude, the extremal S arc that achieves the maximum plane change. In the original statement of the optimal control problem, both the initial and final speed for the constant altitude trajectory were specified. As noted earlier, the initial and final speeds on the S arc are consequences of the relations that hold along the S arc and will not, in general, equal the specified initial and final speeds. By choosing the initial speed consistent with the initial speed on the S arc, we have avoided considering an initial boost or coast arc. Under our assumption that most of the optimal trajectory will lie on a singular hypersurface, we are focusing our attention on the various extremal S arcs that emanate from the initial point on the singular hypersurface; the optimal nonsingular arc leading to this initial point will be the same for each extremal S arc. On the other hand, to allow a meaningful performance comparison between extremal S arcs and ensure that the maximum plane change is a continuously differentiable function of the fuel consumption, we have specified the final speed. In our first numerical example, there were several extremal S arcs for each value of the fuel consumption, each having a different final speed. By specifying the final speed to be less than the smallest of these, we ensured that the final speed could be achieved in each case by following the S arc by a C arc. Combining an extremal S arc with the required final C arc, we obtain an extremal constant altitude trajectory, satisfying all of the boundary conditions, that is, a candidate for maximizing the plane change within the class of constant altitude trajectories satisfying the same physical boundary conditions.

We now address the question, Are any of the extremal S arc/C arc solutions maximizing within the constant altitude class? For each of the extremal S arc/C arc solutions we have computed, we find that the strengthened forms of Goh's necessary conditions are satisfied along the S arc and that the Hamiltonian is maximized over the control set at each point on the C arc. We also note that the necessary condition[10] at the junction of the two arcs is satisfied. Comparing the plane change for neighboring S arc/C arc extremals that satisfy all of the boundary conditions except the transversality condition on B, we find that extremal S arc/C arc trajectories with n odd (where n is as defined earlier) are locally maximizing, whereas those with n even are locally minimizing. For each value of the fuel consumption, the trajectory composed of the S arc, which maximizes the plane change within the class of totally S arc trajectories, and the appropriate C arc yields the largest plane change among the extremal S arc/C arc trajectories. If the constant altitude trajectory that maximizes the plane change is primarily an S arc as we have hypothesized, it appears that we have found the maximizing trajectories for the particular cases we have considered.

Conclusions

Our interest in synergetic plane changes, with a constrained heating rate, led us to consider the problem of maximizing the orbital plane change during constant altitude flight. The problem has been formulated mathematically, and necessary conditions for the optimal trajectories and controls have been derived. In general, the optimal trajectories may be composed of boost, coast, and singular arcs with respect to the thrust control. Our previous results, obtained under more restrictive assumptions, suggested that the maximum plane change at high altitude would be achieved by flying at nearly circular speed. Since this speed can only be sustained by using an intermediate thrust level, we hypothesized that at high altitude the optimal trajectory would consist primarily of a singular arc. For the singular arc, the extremal controls could be determined in feedback form (in terms of the state variables), except for a constant parameter. From this one-parameter family of extremal controls, we determined numerically for two constant altitude cases all of the members of this family that satisfied the final conditions except for that on the speed. The maximizing singular arc was then determined by direct comparison of the achieved plane change. In order to satisfy the final condition on the speed, it was necessary to follow the singular arc with a coast arc. The singular-arc/coast-arc trajectory is an extremal solution for the constant altitude optimal synergetic plane change problem. Along the trajectory, we have verified that the strengthened forms of Goh's necessary conditions are satisfied on the singular arc and that the Hamiltonian is maximized on the coast arc. By numerical perturbation, the singular-arc/coast-arc trajectory was found to locally maximize the plane change. If our assumption that the optimal trajectory consists primarily of a singular arc is correct, it is likely that we have determined the globally optimal trajectory. In any case, we have shown by direct numerical comparison that our solution yields a larger plane change than the optimal steady cruise solution.

The primary physical conclusion of this paper is that, during constant altitude flight, the maximum plane change is not, in general, achieved by flying at a constant angle of attack, i.e., with a constant lift coefficient. The additional propellant consumption incurred by maintaining constant angle of attack grows as the range over which the flight takes place increases. Thus, the advantage of variable angle-of-attack flight increases as the flight altitude increases. For very high altitude flight, which might be chosen to reduce the heating rate, the aerodynamic force is small, and it may take a substantial portion of a revolution, or even several revolutions, to effect the required plane change. In this case, the optimal angle-of-attack control becomes near periodic. On the other hand, the speed, although it has not been constrained to be constant, is nearly constant and approximately circular. Circular speed is advantageous because the vertical force balance required to sustain constant altitude is achieved without aerodynamic lift; the lift can thus be used exclusively to effect the plane change.

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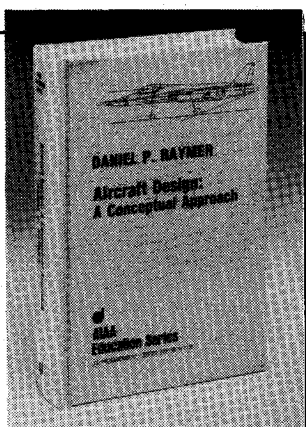

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